Received March 1998
Revised October 1998
Accepted November 1998

Unsteady free convective flow in an enclosure with a stepwise periodically varying side-wall heat flux

P.H. Oosthuizen
Heat Transfer Laboratory, Department of Mechanical Engineering, Queen's University, Kingston, Ontario, Canada

Keywords *Enclosures, Flow, Natural convection*

Abstract *Free convective flow in a square enclosure with one of the vertical walls heated and with the opposite vertical wall cooled to a uniform temperature, the remaining walls being adiabatic, has been numerically studied. The heat flux at the heated wall is spatially uniform but is, in general, varying in a stepwise manner with time. The flow has been assumed to be laminar and two-dimensional. Fluid properties have been assumed constant except for the density change with temperature that gives rise to the buoyancy forces. The governing equations, expressed in terms of stream function and vorticity, have been written in dimensionless form. The resultant equations have been solved using the finite-element method. Because of the possible applications that motivated the study, results have only been obtained for a Prandtl number of 0.7. Results have then been obtained for modified Rayleigh numbers between 1,000 and 1,000,000 for a wide range of dimensionless amplitudes and periods of the heat flux variation.*

Nomenclature

A = dimensionless half amplitude
A' = half amplitude of heat flux variation
g = gravitational acceleration
k = thermal conductivity
n = n'/W'
n' = coordinate measured normal to wall section being considered
P' = period of hot wall temperature fluctuation
P = dimensionless period of hot wall temperature fluctuation
Pr = Prandtl number
q' = heat flux at any instant of time
$\bar{q}'$ = time averaged heat flux
q = $q'/\bar{q}'$
Ra^* = heat flux Rayleigh number based on W'
T = dimensionless temperature
T' = temperature
T'_C = temperature of cold wall
T_m = time-averaged mean dimensionless temperature of heated wall
$T_{\min}$ = minimum mean dimensionless temperature of heated wall
$T_{\max}$ = maximum mean dimensionless temperature of heated wall
t = dimensionless time
t' = time
u = dimensionless velocity component in x direction
u' = velocity component in x' direction
v = dimensionless velocity component in y direction
v' = velocity component in y' direction
W' = width and height of enclosure
x = dimensionless x' coordinate
x' = horizontal coordinate position
y = dimensionless y' coordinate
y_e = value of y' at top of heated wall section
y_s = value of y' at bottom of heated wall section
y' = vertical coordinate position
α = thermal diffusivity
β = bulk coefficient
ν = kinematic viscosity
ψ = dimensionless stream function
ψ' = stream function
ω = dimensionless vorticity
ω' = vorticity

This work was supported by the Natural Sciences and Engineering Research Council of Canada.

© MCB University Press, 0961-5539

Introduction

Most available studies of free convective flow in enclosures have been concerned with steady flow situations and most studies have considered the case where the wall temperatures are specified. In many real situations, however, the flow in the enclosure is not steady and the heat flux rather than the temperature on at least one of the walls is known and is varying periodically with time. In order to provide some results that indicate what heat transfer rates will exist in such cases, a numerical study of free convective flow in a square enclosure with one vertical wall heated, the heat flux varying in a stepwise manner with time on this wall, and with the opposite wall cooled to a uniform lower temperature, T'_C, has been undertaken. The remaining walls are adiabatic. The flow situation considered is, thus, as shown in Figure 1. The flow has been assumed to be laminar and two-dimensional. The situation considered is an approximate model of that occurring in the cooling of electronic and electrical equipment.

The flow and heat transfer in enclosures that results when the temperature of one vertical wall is suddenly increased has been quite widely studied, e.g. see Chan and Banerjee (1979), Patterson and Imberger (1980), Kublbeck *et al.* (1980), Ivey (1984), Nicolett *et al.* (1985), Hall *et al.* (1988), Schladow *et al.* (1989), Hyun and Lee (1989), Patterson and Armfield (1990), Jeevaraj and Patterson (1992), Oosthuizen and Paul (1990), Kuhn and Oosthuizen (1987a; 1987b; 1988). The case where the wall temperature increases linearly with time is discussed by Vasseur and Robillard (1982) and Schladow *et al.* (1989). The case where the hot wall temperature is varying periodically with time has received relatively less attention. A study of the effects of a sinusoidally varying hot wall temperature for the fully heated wall case is described in Kazmierczak and Chinoda (1992) which also gives a review of past work concerned with the time-varying hot wall temperature case. Oosthuizen and Paul (1993) describe a study of the case where the hot wall is only partially heated and where its temperature is again varying sinusoidally with time. Antohe and Lage (1996b)

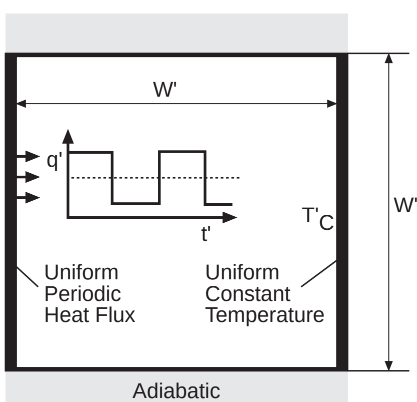

Figure 1.
Situation considered in present study

describe an experimental study of a situation in which the temperature is varying periodically with time. Resonance in enclosures with a periodically varying wall temperature is discussed by Lage and Bejan (1993), Kwak and Hyun (1996), Iwatsu *et al.* (1992), Fu and Shieh (1992), Ho and Chu (1993), Antohe and Lage (1996a) and Kwak *et al.* (1998). A review of past work in this area is given by Kwak and Hyun (1998). Thus, past studies of unsteady flow in an enclosure have mainly been concerned with the case where the hot wall temperature is varying with time. The present study differs from the past work in that it deals with the practically important case where the wall heat flux is varying sharply with time. The study is a direct extension of that described by Oosthuizen and Paul (1996) in which there is a heat flux which varies sinusoidally with time.

Governing equations and solution procedure

It has been assumed that the flow is laminar and two-dimensional and that the fluid properties are constant except for the density change with temperature which gives rise to the buoyancy forces, this having been treated by using the Boussinesq approach.

The solution has been obtained in terms of the stream function and vorticity defined, as usual, by:

$$u' = \frac{\partial \psi'}{\partial y'} \quad , \qquad v' = \frac{\partial \psi'}{\partial x'} \quad ,$$

$$\omega' = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'} \tag{1}$$

The prime (′) denotes a dimensional quantity.

The following dimensionless variables have then been defined:

$$\psi = \psi'/\alpha \ , \quad \omega = \omega' W'^2/\alpha \ , \quad x = x'/W' \ , \quad y = y'/W'$$

$$T = (T' - T'_C)/(\overline{q'}W'/k) \ , \quad t = t'\nu/W'^2 \tag{2}$$

where T'_C is the temperature of the cold wall and $\overline{q'}$ is the mean heat flux rate from the heated wall section.

In terms of these dimensionless variables, the governing equations are:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \tag{3}$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y} - Pr\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) = Ra^* \frac{\partial T}{\partial x} \tag{4}$$

$$\frac{\partial T}{\partial t}+\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}-\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=0 \qquad (5)$$

Here Ra^* is the heat flux Rayleigh number based on the enclosure width, W', i.e.:

$$Ra^* = \beta g \overline{q'} W'^4 / k\nu\alpha \qquad (6)$$

The boundary conditions on the solution are as follows:

On all walls:

$$\psi = 0, \frac{\partial \psi}{\partial n} = 0$$

For $x = 0$:

$$\frac{\partial T}{\partial x} = -q \qquad (7)$$

For $x = 1$:

$$T = 0$$

For $y = 0$ and $y = 1$ for all x:

$$\frac{\partial T}{\partial y} = 0$$

Here $q = q'/\overline{q'}$ is the time varying dimensionless heat flux on the heated wall section, the time-averaged value of q thus being 1.

It has been assumed that the fluid is initially at rest and at the cold wall temperature, i.e. it has been assumed that the initial conditions are:

$$t = 0 : \psi = 0 \;, \quad T = 0 \qquad (8)$$

The above dimensionless equations, subject to the initial and boundary conditions, have been solved using a finite element procedure. An in-house software package has been used. The solution was obtained using linear, triangular elements.

This solution gives, at any instant of dimensionless time, the variation of the local dimensionless temperature on the heated wall. This variation has then been used to find the instantaneous spatially averaged mean temperature of the heated wall section i.e.:

$$\overline{T} = \int_0^1 T dy \qquad (9)$$

Results

The solution, in general, has the following parameters:

- The Rayleigh number, Ra^*.
- The Prandtl number, Pr.
- The dimensionless half amplitude of the hot wall heat flux variation, A.
- The dimensionless period of the hot wall heat flux variation, P. This is defined as follows, P' being the period in seconds:

$$P = P'\alpha / W'^2$$

Because of the possible applications that motivated the study (see Introduction), results are only given here for a Prandtl number of 0.7. Results for other values of Pr show the same basic trends as those given here. Results have been obtained for Rayleigh numbers between 1,000 and 1,000,000 for dimensionless periods between 0.0005 and 0.1 for dimensionless half amplitudes of between 0 and 1. A value of A equal to zero means that there is a constant heat flux at the heated wall section while a value of 1 will give a heat flux that varies between 0 and 2 (see Figure 2).

Figure 3 shows a typical variation of the mean hot wall temperature with dimensionless time for the particular case of $A = 0.5$, $P = 0.01$ and $Ra^* = 10^6$. It will be seen that, since the fluid in the enclosure is at rest when heating begins, there is an initial transient phase but by a dimensionless time of 0.3 the variation has become periodic. The dimensionless time taken for the periodic state to be reached was found to be mainly dependent on the Rayleigh number. Except for the very smallest Rayleigh numbers considered, the periodic state was found to have been reached by a dimensionless time of less than 0.4. The initial transient will not be considered here, attention being given in the rest of this paper to the results in the periodic state, i.e. for dimensionless times of greater than 0.4. The results given in Figure 3 are for a dimensionless amplitude of 0.5. The dimensionless wall heat flux, therefore, varies between 0.5 and 1.5, i.e. the maximum value is three times the minimum value. It will be

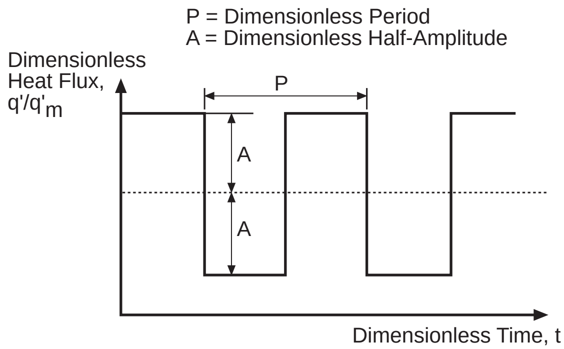

Figure 2.
Hot wall heat flux variation with time considered

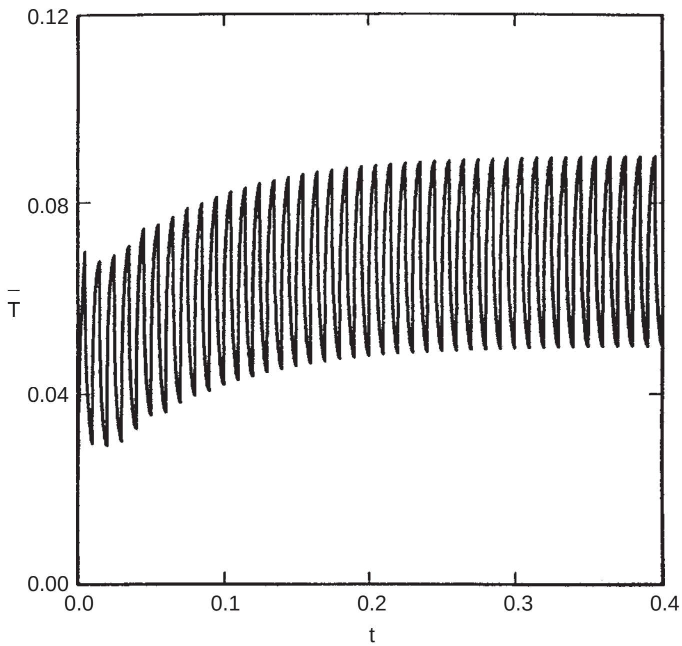

Figure 3. Variation of mean dimensionless hot wall temperature with dimensionless time for P=0.01, A=0.5 and $Ra^*=10^6$

seen from Figure 3 that the ratio of the maximum to the minimum mean wall temperature is much less than this, indicating that a pseudo-steady state flow does not exist in the enclosure in the situation considered.

The effect of the dimensionless amplitude, A, on the mean hot wall dimensionless temperature in the periodic state will first be considered. Figure 4 shows the variations of the maximum and minimum values of the dimensionless hot wall temperature in the periodic state with dimensionless amplitude A. It will be seen that for the conditions considered, the maximum and minimum values of the dimensionless wall temperature are approximately proportional to A. Figure 4 also shows the variation of the time-averaged mean dimensionless hot wall temperature with A. It will be seen that this average value is essentially constant for the conditions considered and it is equal, essentially, to the value that would exist with steady state heat transfer under the same conditions.

The effect of the dimensionless period, P, on the mean hot wall dimensionless temperature in the periodic state will next be considered. Figure 5 shows the variations of the maximum and minimum values of the dimensionless hot wall temperature in the periodic state with dimensionless period, P. It will be seen that for dimensionless periods above about 0.02 these maximum values are approximately constant but that at smaller values of P the difference between the maximum and minimum values decreases with decreasing P. Figure 5 also shows the variation of the time-averaged mean dimensionless hot wall temperature with P. It will be seen that this average

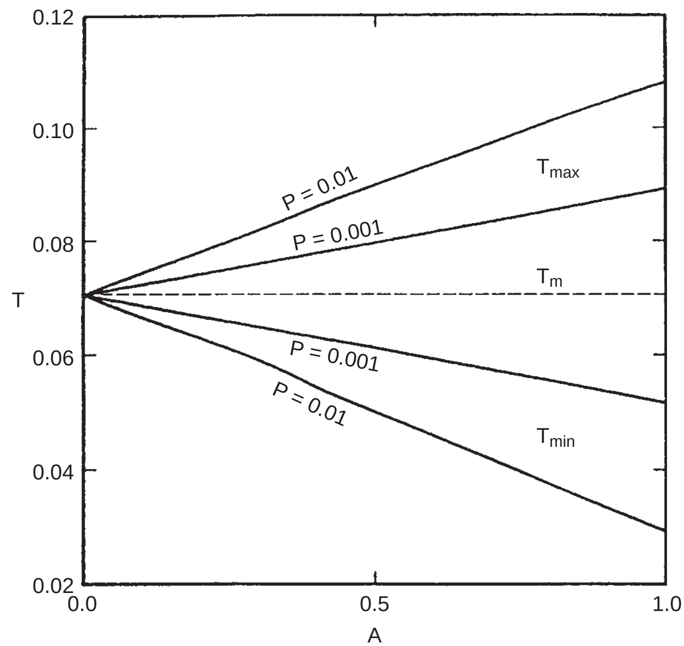

Figure 4.
Effect of A on the maximum and minimum values of mean dimensionless heated wall section temperature for P = 0.01 and P = 0.001 for Ra^* = 0^6

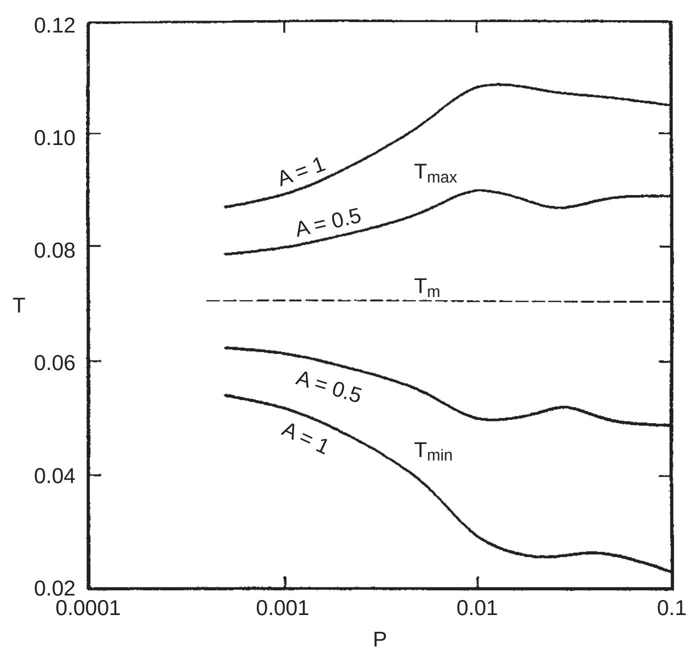

Figure 5.
Effect of P on the maximum and minimum values of mean dimensionless heated wall section temperature for A = 0.5 and A = 1 for Ra^* = 10^6

value is essentially constant for the conditions considered and it is equal, essentially, to the value that would exist with steady state heat transfer under the same conditions.

Lastly, the effect of the Rayleigh number Ra^* on the mean hot wall dimensionless temperature in the periodic state will be considered. Figure 6 shows the variations of the maximum and minimum values of the dimensionless hot wall temperature in the periodic state with modified Rayleigh number Ra^*. At small Rayleigh numbers, when the convective motion has a negligible effect on the heat transfer rate, the heat transfer rate is not dependent on Ra^*. However, for modified Rayleigh numbers above about 1,000, the maximum and minimum values both decrease with increasing Ra^*.

Conclusions

The results obtained in the present study indicate that:

- Unless the Rayleigh number is small (< approximately 1,000), the initial transient resulting from the fact that the fluid is initially at rest becomes negligible for dimensionless times greater than about 0.4, the flow being periodic for dimensionless times greater than this.
- At the smaller values of the dimensionless period, P, here considered, the range over which this dimensionless mean wall temperature varies decreases with decreasing P. At the larger values of P considered, however, the range over which this dimensionless mean wall temperature varies is relatively independent of P.

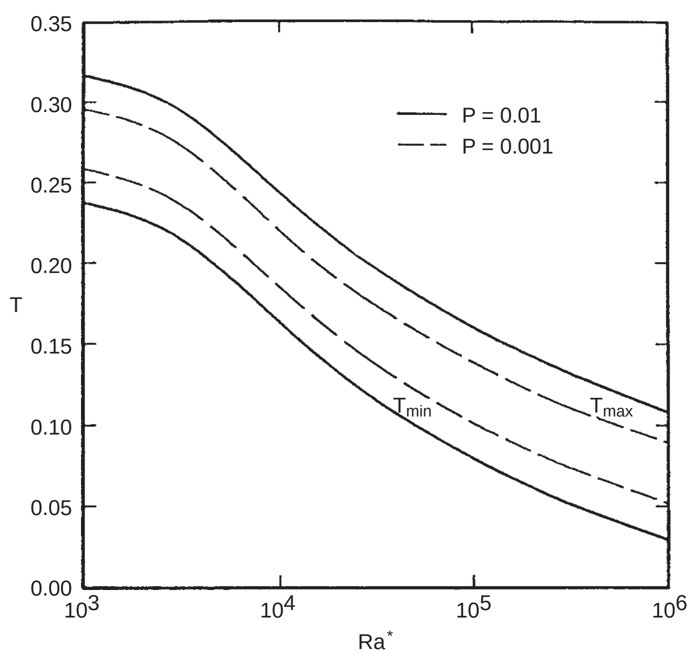

Figure 6.
Effect of Ra^* on the maximum and minimum values of mean dimensionless heated wall section temperature for $A = 1$, $P = 0.01$ and $P = 0.001$

- For the range of parameters covered in the present study, the range over which this dimensionless mean wall temperature varies is approximately proportional to the dimensionless amplitude, A.
- The time-averaged mean dimensionless temperature of the heated wall is, for all conditions considered, essentially equal to the mean wall temperature that would exist at the same Rayleigh number with steady flow in the enclosure.

References

Antohe, B.V. and Lage, J.L. (1996a), "Experimental investigation on pulsating horizontal heating of an enclosure filled with water", *ASME J. Heat Transfer*, Vol. 118, pp. 889-96.

Antohe, B.V. and Lage, J.L. (1996b), "Natural convection in an enclosure under time periodic heating: an experimental study", *ASME Proceedings of the 31st National Heat Transfer Conference*, Vol. 2, HTD-Vol. 324, pp. 57-64.

Chan, A.M.C. and Banerjee, S. (1979), "Three-dimensional numerical analysis of transient natural convection in rectangular enclosures", *J. Heat Transfer*, Vol. 101, pp. 114-19 .

Fu, W.S. and Shieh, W.J. (1992), "A study of thermal convection in an enclosure induced simultaneously by gravity and vibration", *Int. J. Heat Mass Transfer*, Vol. 35, pp. 1695-710.

Hall, J.D., Bejan, A. and Chaddock, J.B. (1988), "Transient natural convection in a rectangular enclosure with one heated side wall", *Int. J. Heat and Fluid Flow*, Vol. 9, pp. 396-404.

Ho, C.J. and Chu, C.H. (1993), "Periodic melting within a square enclosure with an oscillatory surface temperature", *Int. J. Heat Mass Transfer*, Vol. 36, pp. 725-33.

Hyun, J.M. and Lee, J.W. (1989), "Numerical solutions for transient natural convection in a square cavity with different sidewall temperatures", *Int. J. Heat and Fluid Flow*, Vol. 10, pp. 146-51.

Ivey, G.N. (1984), "Experiments on transient natural convection in a cavity", *J. Fluid Mechanics*, Vol. 144, pp. 489-401.

Iwatsu, T., Hyun, J.M. and Kuwahara, K. (1992), "Convection in a differentially-heated square cavity with a torsionally-oscillating lid", *Int. J. Heat Mass Transfer*, Vol. 32, pp. 1069-76.

Jeevaraj, C.G. and Patterson, J.C. (1992), "Experimental study of transient natural convection of glycerol-water mixtures in a side heated cavity", *Int. J. Heat and Mass Transfer*, Vol. 35, pp. 1573-87.

Kazmierczak, M. and Chinoda, Z. (1992), "Buoyancy driven flow in an enclosure with time periodic boundary conditions", *Int. J. Heat and Mass Transfer*, Vol. 35, pp. 1507-18.

Kublbeck, K., Merker, G.P. and Straub, J. (1980), "Advanced numerical computation of two-dimensional time-dependent free convection in cavities", *Int. J. Heat and Mass Transfer*, Vol. 23, pp. 203-17.

Kuhn, D.C.S. and Oosthuizen, P.H. (1987a), "Unsteady natural convection in a partially heated rectangular cavity", *J. Heat Transfer*, Vol. 109 No. 3, pp. 798-801.

Kuhn, D.C.S. and Oosthuizen, P.H. (1987b), "Transient three-dimensional natural convective flow in a rectangular enclosure with two heated elements on a vertical wall", *Proc. 5th Int. Conf. on Numerical Methods in Thermal Problems*, Vol. 1, pp. 524-35.

Kuhn, D.C.S. and Oosthuizen, P.H. (1988), "Transient three-dimensional flow in an enclosure with a hot spot on a vertical wall", *Int. J. Num. Methods in Fluids*, Vol. 8, pp. 369-85.

Kwak, H.S. and Hyun, J.M. (1996), "Natural convection in an enclosure having a vertical side-wall with time-varying temperature", *J. Fluid Mech.*, Vol. 329, pp. 65-88.

Kwak, H.S. and Hyun, J.M. (1998), "Unsteady natural convection in an enclosure", *Heat Transfer 1998: Proceedings 11th International Heat Transfer Conference*, Vol. 1, Korea Society of Mechanical Engineers, Korea, pp. 341-56.

Kwak, H.S., Kuwahara, K. and Hyun, J.M. (1998), "Resonant enhancement of natural convection heat transfer in a square enclosure", *Int. J. Heat Mass Transfer*.

Lage, J.L. and Bejan, A. (1993), "The resonance of natural convection in an enclosure heated periodically from the side", *Int. J. Heat Mass Transfer*, Vol. 36, pp. 2027-38.

Nicolett, V.F., Yang, K.T. and Lloyd, J.R. (1985), "Transient cooling by natural convection in a two-dimensional square enclosure", *Int. J. Heat and Mass Transfer*, Vol. 28, pp. 1721-32.

Oosthuizen, P.H. and Paul, J.T. (1990), "Unsteady free convective flow in an enclosure containing water near its density maximum", in Arpaci, V.S. and Bayazitoglu, Y. (Eds), *Fundamentals of Natural Convection*, Vol. 140, ASME HTD, New York, NY, pp. 83-91.

Oosthuizen, P.H. and Paul, J.T. (1993), "Unsteady natural convective flow in an enclosure with a partially heated wall with a varying temperature", *Proc. 8th Int. Conf. on Numerical Methods in Thermal Problems*, Vol. VIII No. 1, pp. 455-66.

Oosthuizen, P.H. and Paul, J.T. (1996), "Unsteady natural convective flow in an enclosure with a periodically varying side wall heat flux", in Wrobel, L.C., Comini, G., Brebbia, C.A. and Nowak, A.J. (Eds), *Advanced Computational Methods in Heat Transfer IV*, Computational Mechanics Publications, pp. 53-62.

Patterson, J. and Imberger, J. (1980), "Unsteady natural convection in a rectangular cavity", *J. Fluid Mechanics*, Vol. 100 No. 1, pp. 65-86.

Patterson, J.C. and Armfield, S.W. (1990), "Transient features of natural convection in a cavity", *J. Fluid Mechanics*, Vol. 219, pp. 469-97.

Schladow, S.G., Patterson, J.C. and Street, R.L. (1989), "Transient flow in a side-heated cavity at high rayleigh number: a numerical study", *J. Fluid Mechanics*, Vol. 200, pp. 121-48.

Vasseur, P. and Robillard L. (1982), "Natural convection in a rectangular cavity with wall temperature decreasing at a uniform rate", Warme-u. Stoffebertr., Vol. 16, pp. 199-207.